

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Geometry

Similarity and congruence – Mixed problems: Understanding
similarity and congruence

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

Scale factor $20 \frac{20}{8}$ or $\frac{8}{20}$ or 2.5
or $\frac{2}{8}$ or $\frac{8}{4}$ or 4 or $\frac{1}{4}$
oe

5

M1

A1

[2]

Q2.

Side = 4 seen or implied
or 2 – – 2 or 4
or –2 –2 or –4

(a =) 9

M1

A1

Side = 5 seen or implied
or –2 – –7 or 5
or –7 – –2 or –5

(b =) 11

M1

A1

[4]

Q3.

Congruent shape drawn using given side
any orientation

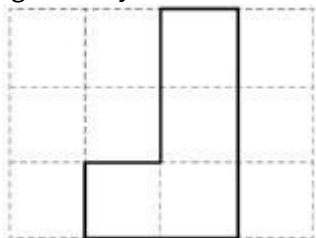
B1

Additional Guidance

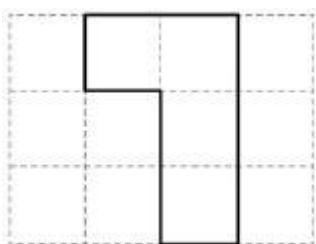
Allow internal lines

Mark intention

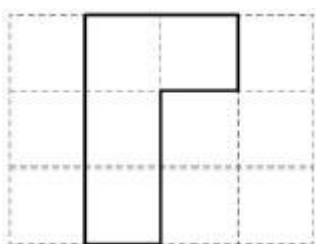
Ignore any labels



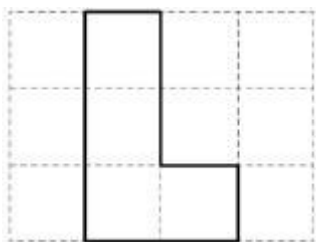
B1



B1



B1



B1

[1]

Q4.

Width = 3 seen or implied
or 2 – –1 or 3
or –1 – 2 or –3

M1

$(a =) 7$

A1

Length = 4 seen or implied
or –2 – –6 or 4
or –6 – –2 or –4

M1

$(b =) 8$

A1

[4]

Q5.

- (a) 2 by 4 or 4 by 2 rectangle drawn
accept overlap with given rectangle
mark intention

B1

Additional Guidance

Ignore shading and internal lines

If more than one shape drawn apply the rules of choice

- (b) Rectangle with dimensions in ratio 1 : 2 or 2 : 1,
but not 2 by 4 or 4 by 2
accept overlap with given rectangle
mark intention

B1

Additional Guidance

Ignore shading and internal lines

If more than one shape drawn apply the rules of choice

[2]

Q6.

(a) $\frac{5}{8}$

oe

B1

(b) **Alternative method 1**

$20 \div 5$ or 4 or $5 \div 20$ or $\frac{1}{4}$

or

$8 \div 5$ or $\frac{8}{5}$ or $5 \div 8$ or $\frac{5}{8}$

oe

M1

32

A1

Alternative method 2

$\frac{20}{y} = \text{their } \frac{5}{8}$

oe

M1

32

ft their cos x from **(a)**

A1ft

Alternative method 3

$\cos^{-1}(\text{their } \frac{5}{8})$ or [51.3, 51.4]

This could be on the diagram or seen in part **(a)**

M1

32

ft their cos x from **(a)**

A1ft

[3]

Q7.

(a) $\frac{3}{4}$

oe

B1

(b) **Alternative method 1**

$6 \div 4$ or 1.5 or $4 \div 6$ or $\frac{2}{3}$
or

$4 \div 3$ or $\frac{4}{3}$ or $3 \div 4$ or $\frac{3}{4}$
oe

M1

4.5

A1

Alternative method 2

$\frac{y}{6}$ = their $\frac{3}{4}$
oe

M1

4.5

ft their tan x from (a)

A1ft

Alternative method 3

$\tan^{-1}$ (their $\frac{3}{4}$) or [36.8, 36.9]
This could be on the diagram or seen in part (a)

M1

4.5

ft their tan x from (a)

A1ft

Additional Guidance

For M1, accept $\frac{2}{3}$ or $\frac{4}{3}$ given as a decimal truncated or rounded to 2dp or better
Award both marks for an answer of 8 in part (b) unless an incorrect statement is made; eg

M0A0

in (a), $\tan x = \frac{4}{3}$, in (b), $\frac{3}{4} = \frac{y}{6}$, answer 4.5

M1A1

in (a), $\tan x = \frac{4}{3}$, in (b), $\tan x = \frac{6}{y}$ (incorrect), $\frac{4}{3} = \frac{6}{y}$, answer 4.5

M0A0

in (a), $\tan x = \frac{4}{3}$, in (b), $\tan x = \frac{y}{6}$, $\frac{4}{3} = \frac{y}{6}$, answer 8

M1A1ft

If the answer line is blank, but 4.5 is seen correctly embedded or as the correct length on the diagram, award only the method mark

M1A0

In alt 2 and alt 3 their tan x must be a value for tan x and not a value for x

[3]

Q8.

Alternative method 1

10 ÷ 4 or 2.5

or 4 ÷ 10 or 0.4

or $\frac{1}{2} \times (18 + 10) \times 25$ or 350
oe

M1

18 ÷ their 2.5

or 18 × their 0.4 or 7.2

or 25 ÷ their 2.5

or 25 × their 0.4 or 10
oe

M1dep

$\frac{1}{2} \times (18 + 10) \times 25$ or 350
and

$\frac{1}{2} \times (\text{their } 7.2 + 4) \times \text{their } 10$ or 56
Must see working

M1dep

350 – 56 = 294

Do not award without working seen

A1

Alternative method 2

10 ÷ 4 or 2.5

or 4 ÷ 10 or 0.4

or $\frac{1}{2} \times (18 + 10) \times 25$ or 350
oe

M1

(Area scale factor =) (their 2.5)²
or (their 0.4)²

M1dep

their 350 ÷ (their 2.5)²
or their 350 × (their 0.4)² or 56

Must see working

M1dep

350 – 56 = 294

Do not award without working seen

A1

[4]

Q9.

Alternative method 1

$$\frac{x}{x+5} = \frac{6}{10}$$

oe

Setting up a correct equation

M1

$$10 \times x = 6 \times (x + 5)$$

oe

Eliminating fractions

M1dep

$$10x - 6x = 30$$

$$\text{or } 4x = 30$$

oe

Collecting terms

M1dep

$$7.5$$

A1

Alternative method 2

$$\frac{x+5}{10} = \frac{5}{4}$$

$$\text{or } \frac{x}{6} = \frac{5}{4}$$

oe

Setting up a correct equation

M1

$$4(x + 5) = 50 \text{ or } 4x + 20 = 50$$

oe

Eliminating fractions

M1dep

$$4x = 50 - 20 \text{ or } 4x = 30$$

oe

Collecting terms

M1dep

$$7.5$$

A1

[4]

Q10.

$$\frac{12}{3} \text{ or } 4$$

$$\text{or } \frac{3}{12} \text{ or } \frac{1}{4}$$

oe

B1

$$\frac{2x-3}{5x} = \frac{3}{12}$$

oe

M1

$$12(2x-3) = 3 \times 5x$$

$$\text{or } 24x - 36 = 15x$$

$$\text{or } 9x = 36$$

$$\text{or } 4(2x-3) = 5x$$

$$\text{or } 8x - 12 = 5x$$

$$\text{or } 3x = 12$$

oe

M1

$$x = 4$$

A1

$$(5 \times \text{their } 4)^2 - 12^2 \text{ or } 256$$

$$\frac{1}{2} \times 4 \times 3 \text{ or } 6$$

M1

$$\sqrt{(5 \times \text{their } 4)^2 - 12^2} \text{ or } 16$$

$$\frac{1}{2} \times 16 \times 12 \text{ or } 6 \times 4^2$$

M1

$$96$$

A1

[7]

Q11.

(a) $\frac{2}{5}$

B1

Alternative method 1

(b) $7 \div \frac{2}{5}$

or $7 \times \frac{5}{2}$ or 17.5

or $\frac{6}{5}$

or $\frac{5}{6}$

oe

$$\frac{?}{6w} = \frac{7}{5w}$$

M1

their $17.5 \times \frac{6}{5}$

or 21

oe

$$7 \times \frac{6w}{5w}$$

M1

$$21 \times \frac{2}{5}$$

or $7 \times \frac{6}{5}$

or 8.4

or $10 + 17.5 + 21$ or 48.5

oe

M1dep

19.4

A1

Alternative method 2

$$5w \times \frac{2}{5} = 7 \text{ or } \frac{5w}{10} = \frac{7}{4} \text{ or } \frac{5w}{7} = \frac{10}{4}$$

oe

M1

$$(w =) \frac{7}{5} \times \frac{5}{2} \text{ or } 3.5$$

oe

M1(Perimeter of A =) $10 + 17.5 + 21$

or 48.5

or (Third side of B =)

$$6 \times 3.5 \times \frac{2}{5}$$

or 8.4

oe

M1

19.4

A1**[5]****Q12.**

$$CN = 5 \text{ or } CB = 10$$

*Check diagram***M1**

$$(AC =) 30 - 13 - 5 - 5 (= 7)$$

$$15 - (11.5)$$

A1

3.5

A1**[3]**

Q13.

Alternative method 1

$48 \div 2 \times 3$ or 72

oe

M1

their 72 $\div 2$ or 36

$$\cos^{-1}\left(\frac{36}{141}\right) \text{ or } 75.2$$

M1dep

$141^2 - \text{their } 36^2$ or 18 585

ft their base $\div 2$

$$\sin(\text{their } 75.2) = \frac{h}{141}$$

$$\text{or } \tan(\text{their } 75.2) = \frac{h}{\text{their } 36}$$

M1dep

$$\sqrt{141^2 - \text{their } 36^2} \text{ or } \sqrt{18\,585}$$

$$141 \times \sin(\text{their } 75.2)$$

$$\text{or their } 36 \times \tan(\text{their } 75.2)$$

M1dep

[136.2, 136.4] or 136

A1

Alternative method 2

$141 \div 3$ or 47

oe

M1

24 and their 47 $\times 2$

or 24 and 94

or 12 and their 47

$$\cos^{-1}\left(\frac{24}{94}\right) \text{ or } 75.2$$

M1dep

their $94^2 - 24^2$ or 8260

$$\text{or } \sqrt{8260}$$

or 90.88...

or their $47^2 - 12^2$ or 2065

$$\text{or } \sqrt{2065}$$

or 45.44...

$$\sin(\text{their } 75.2) = \frac{h}{\text{their } 94}$$

$$\text{or } \tan(\text{their } 75.2) = \frac{h}{24}$$

M1dep

$$\sqrt{\text{their } 94^2 - 24^2} \times 3 \div 2$$

$$\text{or } \sqrt{8260} \times 3 \div 2$$

$$\text{or } 90.88... \times 3 \div 2$$

$$\text{or } \sqrt{\text{their } 47^2 - 12^2} \times 3$$

$$\text{or } \sqrt{2065} \times 3$$

$$\text{or } 45.44... \times 3$$

$$\text{their } 94 \times \sin(\text{their } 75.2) \times 3 \div 2$$

$$\text{or } 24 \times \tan(\text{their } 75.2) \times 3 \div 2$$

M1dep

[136.2, 136.35] or 136

A1

Additional Guidance

Values may be seen on diagram in correct positions

[5]

Q14.

This question has been adapted to better suit the style of the current specification

(a) **Alternative method 1**

$$PR = 8\text{cm}$$

May be on diagram

M1

$$\frac{1}{2} \times 6 \times \text{their } 8$$

M1

$$24$$

A1

Alternative method 2

$$\frac{1}{2} \times 3 \times 4 \text{ or } 6$$

M1

$$\text{their } 6 \times 2^2 \text{ or their } 6 \times 4$$

M1

$$24$$

A1

(b) It is larger than the answer to part (a)

B1

[4]

Q15.

$$\frac{3x}{x} = \frac{36}{x+4} \text{ oe}$$

Scale factor 3 or $\frac{1}{3}$ seen or implied

M1

$$3x(x+4) = 36x \text{ oe}$$

$$36 \div 3 (= 12)$$

M1

$$3(x+4) = 36 \text{ oe}$$

their 12 - 4

$$\text{or } 3x^2 + 12x = 36x$$

M1

$$3x + 12 = 36$$

$$(x =) 8$$

$$\text{or } x + 4 = 12$$

$$\text{or } x = 8$$

or their 8×3

$$\text{or } 3x^2 - 24x = 0$$

$$\text{or } 3x^2 = 24x$$

M1

$$(3x =) 24$$

$$24$$

A1

[5]

Q16.

1.5 or $\frac{2}{3}$ seen

or $\frac{1}{2}$ seen as a scale factor

oe

12 : 8

8 : 12

$\tan C = \frac{8}{11}$ or 36°

$\frac{12}{EC} = \frac{8}{11}$ or $\frac{EC}{12} = \frac{11}{8}$ or $\frac{11 \times 12}{8}$

M1

11×1.5 or $11 \times \frac{1}{2}$

$\frac{1}{2} \times 11 \times 8 \times 1.5^2$

oe

$CE = \frac{12}{\tan(\text{their } 36)}$

M1dep

16.5 or 5.5

99

16.5(...) or 5.5(...)

A1

$\frac{1}{2} (8 + 12) \times \text{their } 5.5$

or

$\frac{1}{2} (8 + 12) \times \text{their } ED$

their 99 - $\frac{1}{2} \times 11 \times 8$

$\frac{1}{2} \times \text{their } 16.5 \times 12 - \frac{1}{2} \times 11 \times 8$

$\text{their } ED \times 8 + \frac{1}{2} \times \text{their } ED \times 4$

M1

55

A1

[5]

Q17.

(a) $\frac{3+x}{4} = \frac{9}{5}$

B1

(b) $9(3+x) = 4 \times 5$

$$3+x = 4 \times \frac{5}{9}$$

M1

$$27 + 9x = 20$$

$$3+x = \frac{20}{9}$$

M1dep

$$9x = 20 - 27$$

$$x = \frac{20}{9} - 3$$

M1dep

$$-\frac{7}{9} \text{ or } -0.77... \text{ or } -0.78$$

oe

A1

Alternative Method 1

$$4(3+x) = 5 \times 9$$

$$3+x = 9 \times \frac{5}{4}$$

M1

$$12 + 4x = 45$$

$$3+x = \frac{45}{4}$$

M1dep

$$4x = 45 - 12$$

$$x = \frac{45}{4} - 3$$

M1dep

$$\frac{33}{4} \text{ or } 8.25$$

oe

A1

Alternative Method 2

$$4(3+x) = 5 \times 9$$

$$3+x = 5 \times \frac{9}{4}$$

M1

$$12 + 4x = 45$$

$$3+x = \frac{45}{4}$$

M1dep

$$4x = 45 - 12$$

$$x = \frac{45}{4} - 3$$

M1dep

$$\frac{33}{4} \text{ or } 8.25$$

oe

A1

Alternative Method 3

$$5(3 + x) = 4 \times 9$$

$$3 + x = 4 \times \frac{9}{5}$$

M1

$$15 + 5x = 36$$

$$3 + x = \frac{36}{5}$$

M1dep

$$5x = 36 - 15$$

$$x = \frac{36}{5} - 3$$

M1dep

$$\frac{21}{5} \text{ or } 4.2$$

oe

A1**Additional Guidance**

$$\frac{3}{4} + \frac{x}{4} = \frac{9}{5}$$

M1

$$\frac{x}{4} = \frac{9}{5} - \frac{3}{4}$$

M1dep

$$x = \left(\frac{9}{5} - \frac{3}{4} \right) \times 4$$

M1dep

$$x = \frac{21}{5}$$

A1

If students use a different equation than that chosen in part (a) they can only score a maximum of 3 out of 4, unless they choose the correct equation

[5]

Q18.

Scale factor $\frac{18}{8}$ or $\frac{8}{18}$ seen oe
11.25 may be on diagram

B1

or $AC = 5 \times 2.25 (= 11.25)$

or angle $B =$ angle E seen

or angle $A =$ angle D seen

Use of cosine rule to work out any angle

$$8^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos C$$

$$18^2 = 9^2 + \text{their } 11.25^2 - 2 \times 9 \times \text{their } 11.25 \times \cos C$$

$$4^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos D$$

$$9^2 = 18^2 + \text{their } 11.25^2 - 2 \times 18 \times \text{their } 11.25 \times \cos A$$

$$5^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \times \cos E$$

$$\text{their } 11.25^2 = 9^2 + 18^2 - 2 \times 9 \times 18 \times \cos B$$

M1

Correct rearranging of formula to isolate cosine

$$\frac{4^2 + 5^2 - 8^2}{2 \times 4 \times 5} \text{ or } -\frac{23}{40}$$

$$\frac{9^2 + \text{their } 11.25^2 - 18^2}{2 \times 9 \times \text{their } 11.25} \text{ or } -\frac{23}{40}$$

$$\frac{5^2 + 8^2 - 4^2}{2 \times 5 \times 8} \text{ or } \frac{73}{80}$$

$$\frac{\text{their } 11.25^2 + 18^2 - 9^2}{2 \times \text{their } 11.25 \times 18} \text{ or } \frac{73}{80}$$

$$\frac{4^2 + 8^2 - 5^2}{2 \times 4 \times 8} \text{ or } \frac{55}{64}$$

$$\frac{9^2 + 18^2 - \text{their } 11.25^2}{2 \times 9 \times 18} \text{ or } \frac{55}{64}$$

M1dep

Obtaining one angle

eg $C = 125.(...) \text{ or } 125$

$B = 30.(...) \text{ or } 31 = E$

$A = 24.(...) \text{ or } 24 = D$

May be seen on diagram

A1

Substitution into $\frac{1}{2} ab \sin C$

$$\frac{1}{2} \times 11.25 \times 9 \times \sin 125$$

$$\frac{1}{2} \times 11.25 \times 18 \times \sin 24$$

$$\frac{1}{2} \times 18 \times 9 \times \sin 31$$

or

$$\frac{1}{2} \times 4 \times 5 \times \sin 125$$

$$\frac{1}{2} \times 5 \times 8 \times \sin 24$$

$$\frac{1}{2} \times 4 \times 8 \times \sin 31$$

[41, 42]

M1

A1

[6]